Feature constraints to modelise Unix filesystems

Nicolas Jeannerod

IRIF

February 7, 2018

The CoLiS Project

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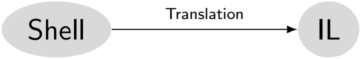

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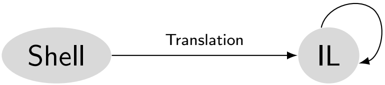

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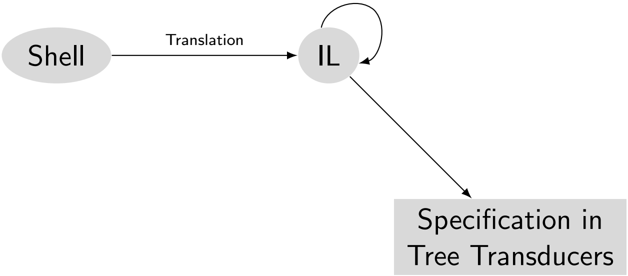

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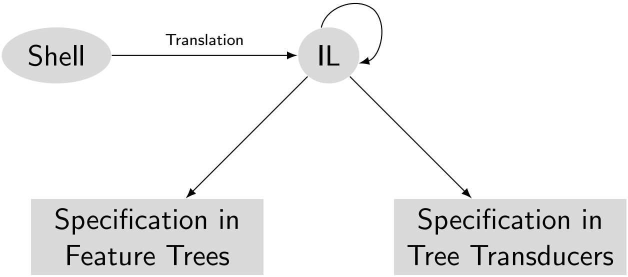

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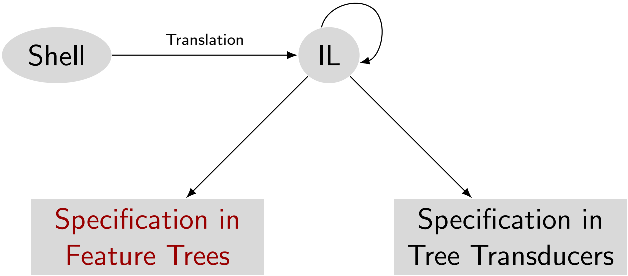

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1. Description of filesystems
   Unix filesystems
   Static description
   Directory update

2. Constraints
   Definitions
   Basic constraints
   Negation

3. Usages
   Decidability of the First-Order Theory
   Automated Specification for Scripts: Proof of Concept

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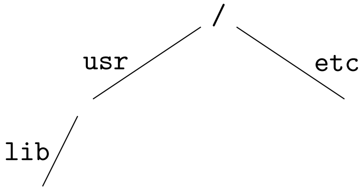

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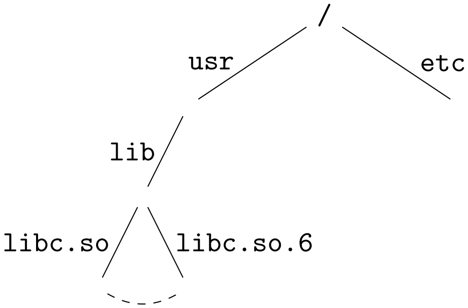

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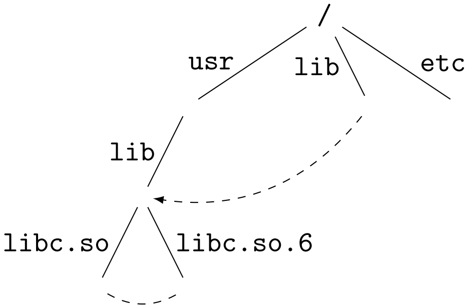

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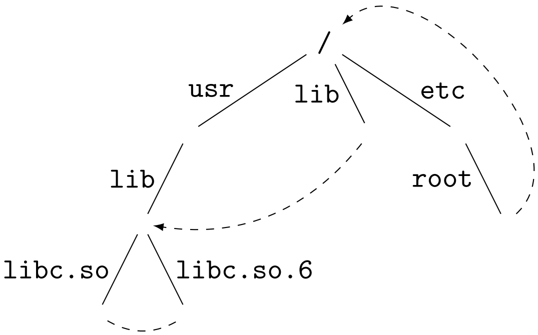

- Basically a tree with labelled nodes and edges;
- There can be sharing at the leafs (hard link between files);
- There can be pointers to other parts of the tree (symbolic links) which may form cycles.

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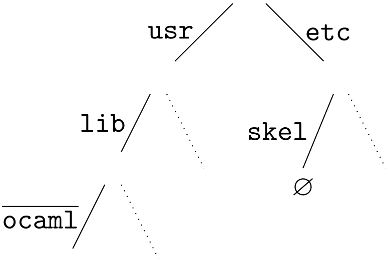

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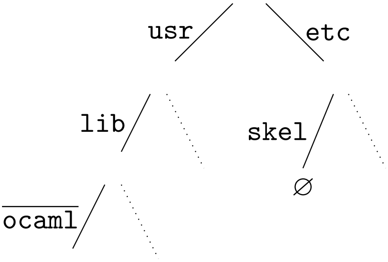

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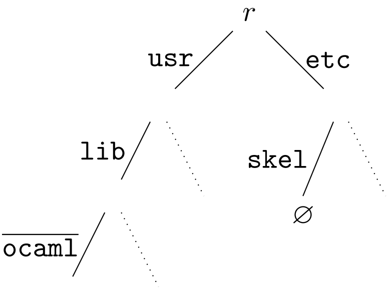

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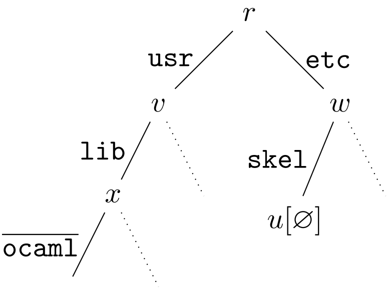

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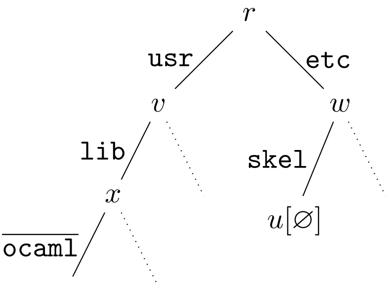

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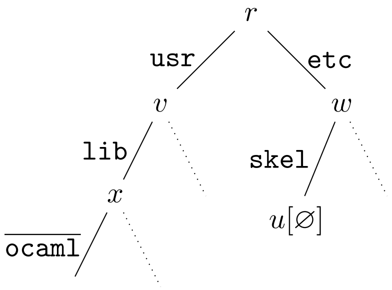

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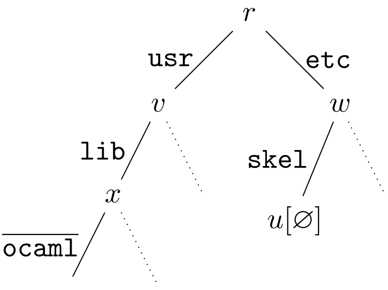

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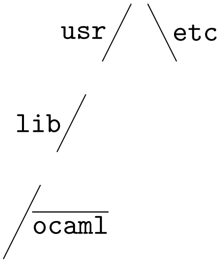

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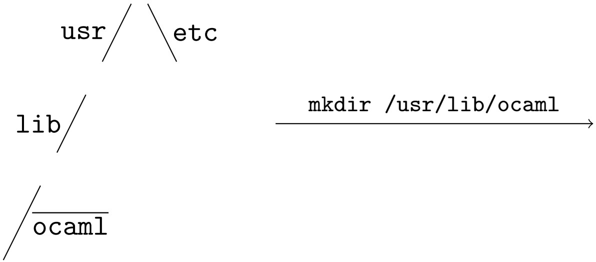

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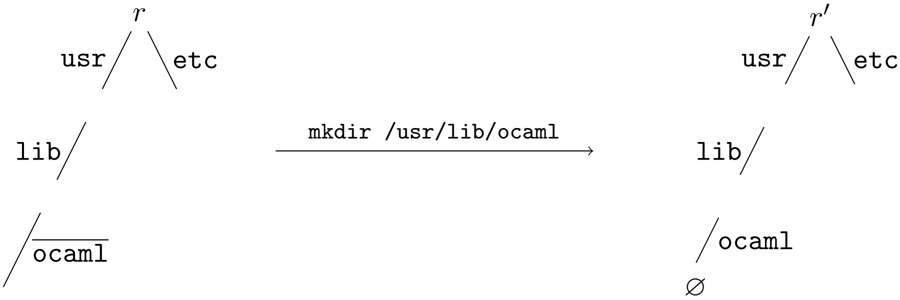

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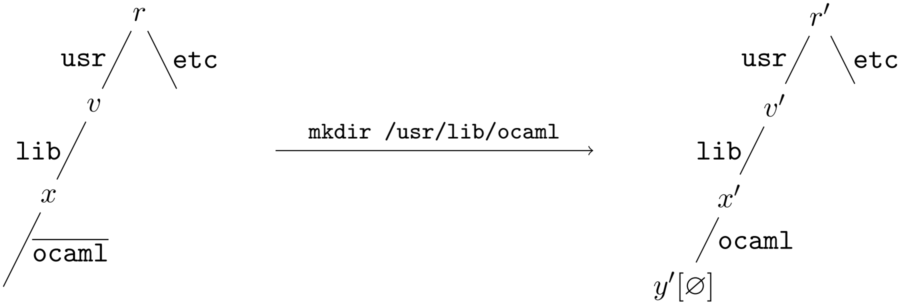

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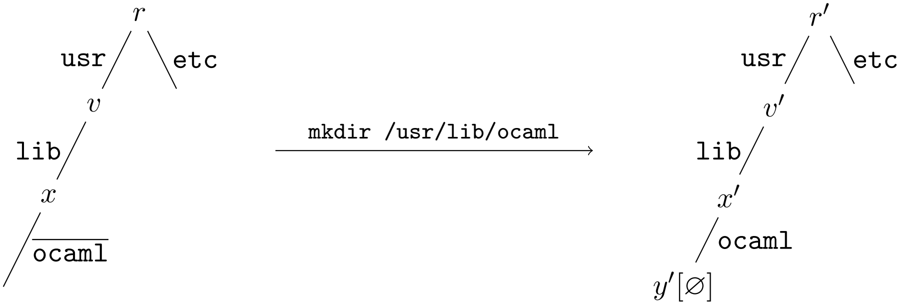

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- Infinite set of variables x, y, etc.;
- $f \in \mathcal{F}$, $F \subset \mathcal{F}$ finite.

Equality	$x \doteq y$		
Feature	$x[f]y$	$x[f]\uparrow$	Absence
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- Wanted: (un)satisfiability of these constraints;
- Bonus point for incremental procedures.

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Feature:	$\mathcal{T}, \rho$	$\models$	$x[f]y$	if	$\rho(x)(f) = \rho(y)$
Absence:	$\mathcal{T}, \rho$	$\models$	$x[f]\uparrow$	if	$f \notin \texttt{dom}(\rho(x))$

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Equality:	$\mathcal{T}, \rho$	$\models$	$x \doteq y$	if	$\rho(x) = \rho(y)$
Feature:	$\mathcal{T}, \rho$	$\models$	$x[f]y$	if	$\rho(x)(f) = \rho(y)$
Absence:	$\mathcal{T}, \rho$	$\models$	$x[f]\uparrow$	if	$f \notin \mathtt{dom}(\rho(x))$
Fence:	$\mathcal{T}, \rho$	$\models$	$x[F]$	if	$\mathtt{dom}(\rho(x)) \subseteq F$

Semantics

$$\mathcal{T}, \rho \models c$$

- $\mathcal{T}$ the model of all feature trees;
- $\rho : \mathcal{V}(c) \to \mathcal{T}$;

Equality:	$\mathcal{T}, \rho$	$\models$	$x \doteq y$	if	$\rho(x) = \rho(y)$
Feature:	$\mathcal{T}, \rho$	$\models$	$x[f]y$	if	$\rho(x)(f) = \rho(y)$
Absence:	$\mathcal{T}, \rho$	$\models$	$x[f]\uparrow$	if	$f \notin \mathtt{dom}(\rho(x))$
Fence:	$\mathcal{T}, \rho$	$\models$	$x[F]$	if	$\mathtt{dom}(\rho(x)) \subseteq F$
Similarity:	$\mathcal{T}, \rho$	$\models$	$x \mathrel{\dot{\sim}_F} y$	if	$\rho(x) \restriction \overline{F} = \rho(y) \restriction \overline{F}$

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Game plan

- Rewriting system;

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Game plan

- Rewriting system;
- Puts constraints in normal form (not necessarily unique);
- Respects equivalences;
- Normal forms: either $\bot$ or with nice properties.

Basic rewriting system

$$
\begin{array}{ll}
x_1[f_1]x_2 \wedge \ldots \wedge x_n[f_n]x_1 & (n \geq 1) \\
x[f]y \wedge x[f]\uparrow & \\
x[f]y \wedge x[F] & (f \notin F)
\end{array}
$$

Clash Patterns

Basic rewriting system

$$
\begin{array}{ll}
x_1[f_1]x_2 \wedge \ldots \wedge x_n[f_n]x_1 & (n \geq 1) \\
x[f]y \wedge x[f]\uparrow & \\
x[f]y \wedge x[F] & (f \notin F)
\end{array}
$$

Clash Patterns

$$
\begin{array}{rcll}
\exists X, x \cdot (x \doteq y \wedge c) & \Rightarrow & \exists X \cdot c\{x \mapsto y\} & (x \neq y) \\
\exists X, z \cdot (x[f]y \wedge x[f]z \wedge c) & \Rightarrow & \exists X \cdot (x[f]y \wedge c\{z \mapsto y\}) & (y \neq z) \\
x \mathrel{\dot\sim}_F y \wedge x \mathrel{\dot\sim}_G y \wedge c & \Rightarrow & x \mathrel{\dot\sim}_{F \cap G} y \wedge c &
\end{array}
$$

Simplification Rules

Basic rewriting system

$$x_1[f_1]x_2 \wedge \ldots \wedge x_n[f_n]x_1 \qquad (n \geq 1)$$
$$x[f]y \wedge x[f]\uparrow$$
$$x[f]y \wedge x[F] \qquad (f \notin F)$$

Clash Patterns

$$\exists X, x \cdot (x \doteq y \wedge c) \quad \Rightarrow \quad \exists X \cdot c\{x \mapsto y\} \qquad (x \neq y)$$
$$\exists X, z \cdot (x[f]y \wedge x[f]z \wedge c) \quad \Rightarrow \quad \exists X \cdot (x[f]y \wedge c\{z \mapsto y\}) \qquad (y \neq z)$$
$$x \mathrel{\dot{\sim}_F} y \wedge x \mathrel{\dot{\sim}_G} y \wedge c \quad \Rightarrow \quad x \mathrel{\dot{\sim}_{F \cap G}} y \wedge c$$

Simplification Rules

$$x \mathrel{\dot{\sim}_F} y \wedge x[f]z \wedge c \quad \Rightarrow \quad x \mathrel{\dot{\sim}_F} y \wedge x[f]z \wedge y[f]z \wedge c \qquad (f \notin F)$$
$$x \mathrel{\dot{\sim}_F} y \wedge x[f]\uparrow \wedge c \quad \Rightarrow \quad x \mathrel{\dot{\sim}_F} y \wedge x[f]\uparrow \wedge y[f]\uparrow \wedge c \qquad (f \notin F)$$
$$x \mathrel{\dot{\sim}_F} y \wedge x[G] \wedge c \quad \Rightarrow \quad x \mathrel{\dot{\sim}_F} y \wedge x[G] \wedge y[F \cup G] \wedge c$$
$$x \mathrel{\dot{\sim}_F} y \wedge x \mathrel{\dot{\sim}_G} z \wedge c \quad \Rightarrow \quad x \mathrel{\dot{\sim}_F} y \wedge x \mathrel{\dot{\sim}_G} z \wedge y \mathrel{\dot{\sim}_{F \cup G}} z \wedge c$$
$$(\text{if } \textstyle\bigcap_{y \dot{\sim}_H z} H \not\subseteq F \cup G)$$

Propagation Rules

Properties

Lemma

The basic constraint system terminates and yields a clause that is equivalent to the first one.

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- *c is in normal form;*

Properties

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The basic constraint system terminates and yields a clause that is equivalent to the first one.

Lemma

Let c be a clause $c = g_c \wedge \exists X \cdot l_c$ such that:

- *c is in normal form;*
- *$\mathcal{V}(g_c) \cap X = \varnothing$;*
- *every literal in l_c is about X;*

Properties

Lemma

The basic constraint system terminates and yields a clause that is equivalent to the first one.

Lemma

Let c *be a clause* $c = g_c \wedge \exists X \cdot l_c$ *such that:*

- c *is in normal form;*
- $\mathcal{V}(g_c) \cap X = \varnothing$*;*
- *every literal in* l_c *is about* X*;*
- *there is no* $y[f]x$ *with* $x \in X$ *and* $y \notin X$*.*

Properties

Lemma

The basic constraint system terminates and yields a clause that is equivalent to the first one.

Lemma

Let c be a clause $c = g_c \wedge \exists X \cdot l_c$ such that:

- *c is in normal form;*
- *$\mathcal{V}(g_c) \cap X = \varnothing$;*
- *every literal in l_c is about X;*
- *there is no $y[f]x$ with $x \in X$ and $y \notin X$.*

Then c is equivalent to g_c.

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Negation: new players, new rules

aka *La Slide de la Mort*

Negation: new players, new rules

$$\begin{array}{rcl} \neg x[f]y \wedge c & \Rightarrow & (x[f]\uparrow \vee \exists z \cdot (x[f]z \wedge y \not\sim_{\varnothing} z)) \wedge c \\ \neg x[f]\uparrow \wedge c & \Rightarrow & \exists z \cdot x[f]z \wedge c \end{array}$$

Simple Replacement Rules

Negation: new players, new rules

$$\begin{aligned}
\neg x[f]y \wedge c &\Rightarrow (x[f]\uparrow \vee \exists z \cdot (x[f]z \wedge y \not\sim_{\varnothing} z)) \wedge c \\
\neg x[f]\uparrow \wedge c &\Rightarrow \exists z \cdot x[f]z \wedge c
\end{aligned}$$

Simple Replacement Rules

$$\begin{aligned}
x[F] \wedge \neg x[G] \wedge c &\Rightarrow x[F] \wedge x\langle F \setminus G\rangle \wedge c \\
x[F] \wedge x \not\sim_G y \wedge c &\Rightarrow x[F] \wedge \left(\neg y[F \cup G] \vee x \dot{\neq}_{F\setminus G} y\right) \wedge c \\
x \dot{\sim}_F y \wedge x \not\sim_G y \wedge c &\Rightarrow x \dot{\sim}_F y \wedge x \dot{\neq}_{F\setminus G} y \wedge c
\end{aligned}$$

More Replacement Rules

Negation: new players, new rules

$$\begin{array}{rcl} x[F] \wedge \neg x[G] \wedge c & \Rightarrow & x[F] \wedge x\langle F \setminus G\rangle \wedge c \\ x[F] \wedge x \not\sim_G y \wedge c & \Rightarrow & x[F] \wedge \big(\neg y[F \cup G] \vee x \dot{\neq}_{F\setminus G}\, y\big) \wedge c \\ x \dot{\sim}_F y \wedge x \not\sim_G y \wedge c & \Rightarrow & x \dot{\sim}_F y \wedge x \dot{\neq}_{F\setminus G}\, y \wedge c \end{array}$$

More Replacement Rules

Negation: new players, new rules

$$
\begin{array}{rcl}
x[F] \wedge \neg x[G] \wedge c & \Rightarrow & x[F] \wedge x\langle F \setminus G\rangle \wedge c \\
x[F] \wedge x \not\sim_G y \wedge c & \Rightarrow & x[F] \wedge \left(\neg y[F \cup G] \vee x \dot{\neq}_{F\setminus G}\, y\right) \wedge c \\
x \dot{\sim}_F\, y \wedge x \not\sim_G y \wedge c & \Rightarrow & x \dot{\sim}_F\, y \wedge x \dot{\neq}_{F\setminus G}\, y \wedge c
\end{array}
$$

More Replacement Rules

Negation: new players, new rules

$$x\langle F\rangle := \bigvee_{f\in F} \exists z \cdot x[f]z$$

$$\begin{array}{rcl} x[F] \wedge \neg x[G] \wedge c & \Rightarrow & x[F] \wedge x\langle F \setminus G\rangle \wedge c \\ x[F] \wedge x \not\sim_G y \wedge c & \Rightarrow & x[F] \wedge \left(\neg y[F \cup G] \vee x \dot{\neq}_{F\setminus G}\ y\right) \wedge c \\ x \dot{\sim}_F y \wedge x \not\sim_G y \wedge c & \Rightarrow & x \dot{\sim}_F y \wedge x \dot{\neq}_{F\setminus G}\ y \wedge c \end{array}$$

More Replacement Rules

Negation: new players, new rules

$$x \doteq\not_F y := \bigvee_{f \in F} \left(\begin{array}{c} \exists z' \cdot (x[f] \uparrow \wedge y[f]z') \vee \exists z \cdot (x[f]z \wedge y[f] \uparrow) \\ \vee \exists z, z' \cdot (x[f]z \wedge y[f]z' \wedge z \not\sim_\varnothing z') \end{array} \right)$$

$$\begin{array}{rcl} x[F] \wedge \neg x[G] \wedge c & \Rightarrow & x[F] \wedge x\langle F \setminus G \rangle \wedge c \\ x[F] \wedge x \not\sim_G y \wedge c & \Rightarrow & x[F] \wedge \left(\neg y[F \cup G] \vee x \doteq\not_{F \setminus G} y\right) \wedge c \\ x \dot\sim_F y \wedge x \not\sim_G y \wedge c & \Rightarrow & x \dot\sim_F y \wedge x \doteq\not_{F \setminus G} y \wedge c \end{array}$$

More Replacement Rules

Negation: new players, new rules

$$x \dot{\neq}_F\, y := \bigvee_{f \in F} \left(\begin{array}{c} \exists z' \cdot (x[f]\uparrow \wedge y[f]z') \vee \exists z \cdot (x[f]z \wedge y[f]\uparrow) \\ \vee \exists z, z' \cdot (x[f]z \wedge y[f]z' \wedge z \not\sim_{\varnothing} z') \end{array} \right)$$

$$\begin{array}{rcl} x[F] \wedge \neg x[G] \wedge c & \Rightarrow & x[F] \wedge x\langle F \setminus G \rangle \wedge c \\ x[F] \wedge x \not\sim_G y \wedge c & \Rightarrow & x[F] \wedge \big(\neg y[F \cup G] \vee x \dot{\neq}_{F \setminus G}\, y\big) \wedge c \\ x \dot{\sim}_F\, y \wedge x \not\sim_G y \wedge c & \Rightarrow & x \dot{\sim}_F\, y \wedge x \dot{\neq}_{F \setminus G}\, y \wedge c \end{array}$$

More Replacement Rules

$$\begin{array}{rcll} x \dot{\sim}_F\, y \wedge \neg x[G] \wedge c & \Rightarrow & x \dot{\sim}_F\, y \wedge (\neg x[F \cup G] \vee x\langle F \setminus G \rangle) \wedge c & (F \not\subseteq G \\ x \dot{\sim}_F\, y \wedge \neg x[G] \wedge c & \Rightarrow & x \dot{\sim}_F\, y \wedge \neg x[G] \wedge \neg y[G] \wedge c & (F \subseteq G \\ x \dot{\sim}_F\, y \wedge x \not\sim_G z \wedge c & \Rightarrow & x \dot{\sim}_F\, y \wedge \big(x \not\sim_{F \cup G} z \vee x \dot{\neq}_{F \setminus G}\, z\big) \wedge c & (F \not\subseteq G \\ x \dot{\sim}_F\, y \wedge x \not\sim_G z \wedge c & \Rightarrow & x \dot{\sim}_F\, y \wedge x \not\sim_G z \wedge y \not\sim_G z \wedge c & (F \subseteq G \end{array}$$

Enlargement and Propagation Rules

Negation: new players, new rules

$$x \dot{\neq}_F y := \bigvee_{f \in F} \left(\begin{array}{c} \exists z' \cdot (x[f] \uparrow \wedge y[f] z') \vee \exists z \cdot (x[f] z \wedge y[f] \uparrow) \\ \vee \exists z, z' \cdot (x[f] z \wedge y[f] z' \wedge z \not\sim_{\varnothing} z') \end{array} \right)$$

$$\begin{array}{rcl} x[F] \wedge \neg x[G] \wedge c & \Rightarrow & x[F] \wedge x\langle F \setminus G \rangle \wedge c \\ x[F] \wedge x \not\sim_G y \wedge c & \Rightarrow & x[F] \wedge \big(\neg y[F \cup G] \vee x \dot{\neq}_{F \setminus G} y \big) \wedge c \\ x \dot{\sim}_F y \wedge x \not\sim_G y \wedge c & \Rightarrow & x \dot{\sim}_F y \wedge x \dot{\neq}_{F \setminus G} y \wedge c \end{array}$$

More Replacement Rules

Negation: new players, new rules

$$x \dot{\neq}_F \; y := \bigvee_{f \in F} \left(\begin{array}{c} \exists z' \cdot (x[f] \uparrow \wedge y[f]z') \vee \exists z \cdot (x[f]z \wedge y[f] \uparrow) \\ \vee \exists z, z' \cdot (x[f]z \wedge y[f]z' \wedge z \not\sim_{\varnothing} z') \end{array} \right)$$

$$\begin{array}{rcl} x[F] \wedge \neg x[G] \wedge c & \Rightarrow & x[F] \wedge x\langle F \setminus G \rangle \wedge c \\ x[F] \wedge x \not\sim_G y \wedge c & \Rightarrow & x[F] \wedge \left(\neg y[F \cup G] \vee x \dot{\neq}_{F \setminus G} \; y\right) \wedge c \\ x \dot{\sim}_F \; y \wedge x \not\sim_G y \wedge c & \Rightarrow & x \dot{\sim}_F \; y \wedge x \dot{\neq}_{F \setminus G} \; y \wedge c \end{array}$$

More Replacement Rules

Negation: new players, new rules

$$x \dot{\neq}_F y := \bigvee_{f \in F} \left(\begin{array}{c} \exists z' \cdot (x[f] \uparrow \wedge y[f]z') \vee \exists z \cdot (x[f]z \wedge y[f] \uparrow) \\ \vee \exists z, z' \cdot (x[f]z \wedge y[f]z' \wedge z \not\sim_{\varnothing} z') \end{array} \right)$$

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More Replacement Rules

$x[F]$ = "x has no feature outside F"
$x \not\sim_G y$ = "there is a feature outside G that differentiates x and y"

Negation: new players, new rules

$$x \dot{\neq}_F y := \bigvee_{f \in F} \left(\begin{array}{c} \exists z' \cdot (x[f]\uparrow \wedge y[f]z') \vee \exists z \cdot (x[f]z \wedge y[f]\uparrow) \\ \vee \exists z, z' \cdot (x[f]z \wedge y[f]z' \wedge z \not\sim_{\varnothing} z') \end{array} \right)$$

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$x[F]$ = "x has no feature outside F"

$x \not\sim_G y$ = "there is a feature outside G that differentiates x and y"

- either it is in F,
- or it is not,

Negation: new players, new rules

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$x[F] =$ "x has no feature outside F"

$x \not\sim_G y =$ "there is a feature outside G that differentiates x and y"

- either it is in F, and we can list all the cases;
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Negation: new players, new rules

$$x \dot{\neq}_F\ y := \bigvee_{f \in F} \left(\begin{array}{c} \exists z' \cdot (x[f] \uparrow \wedge y[f]z') \vee \exists z \cdot (x[f]z \wedge y[f] \uparrow) \\ \vee \exists z, z' \cdot (x[f]z \wedge y[f]z' \wedge z \not\sim_{\varnothing} z') \end{array} \right)$$

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More Replacement Rules

$x[F]$ = "x has no feature outside F"

$x \not\sim_G y$ = "there is a feature outside G that differentiates x and y"

- either it is in F, and we can list all the cases;
- or it is not, and since $x[F]$ then $\neg y[F \cup G]$.

Properties

Lemma

The constraint system terminates and yields a clause that is equivalent to the first one.

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Let c be a clause $c = g_c \wedge \exists X \cdot l_c$ such that:

- *c is in normal form;*
- $\mathcal{V}(g_c) \cap X = \varnothing$;
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- *there is no $y[f]x$ with $x \in X$ and $y \notin X$.*

Then c is equivalent to g_c.

Does that even terminate?

R-NSim-Fence:

$$x[F] \land x \not\sim_G y \land c$$
$$\Rightarrow \quad x[F] \land \big(\neg y[F \cup G] \lor x \dot{\neq}_{F \setminus G}\, y\big) \land c$$

Does that even terminate?

R-NSim-Fence (for $F = \{f\}$ and $G = \varnothing$):

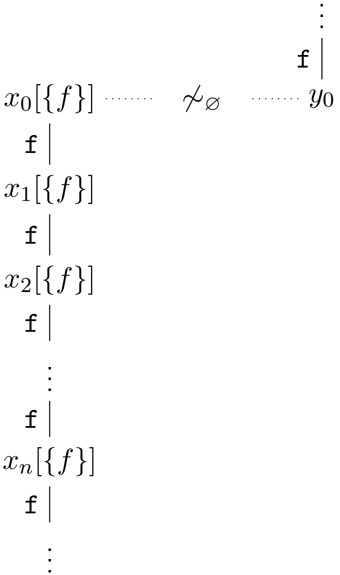
$$x[\{f\}] \wedge x \not\sim_{\varnothing} y \wedge c$$
$$\Rightarrow \quad x[\{f\}] \wedge (\neg y[\{f\}] \vee x \mathrel{\dot{\neq}_f} y) \wedge c$$

Does that even terminate?

R-NSim-Fence (for $F = \{f\}$ and $G = \varnothing$):

$$x[\{f\}] \wedge x \not\sim_\varnothing y \wedge c$$
$$\Rightarrow \quad \exists z, z' \cdot x[f]z \wedge y[f]z' \wedge z \not\sim_\varnothing z' \wedge x[\{f$$

Does that even terminate?



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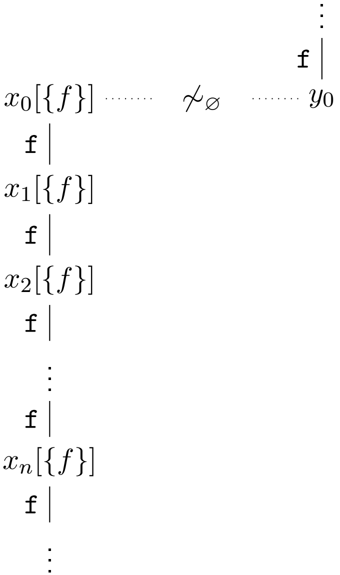

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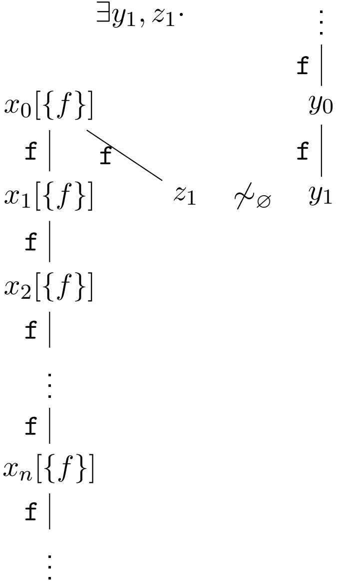
- R-NSIM-FENCE with x_0 and y_0;



R-NSim-Fence (for $F = \{f\}$ and $G = \varnothing$):

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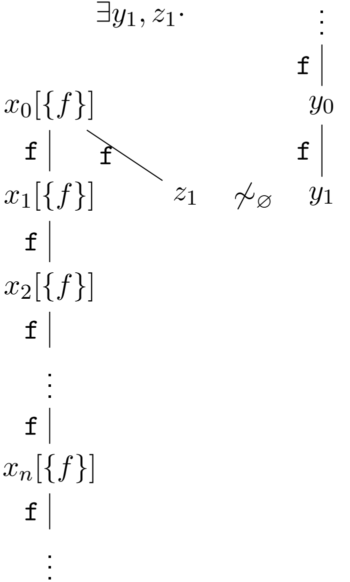

R-NSIM-FENCE (for $F = \{f\}$ and $G = \varnothing$):

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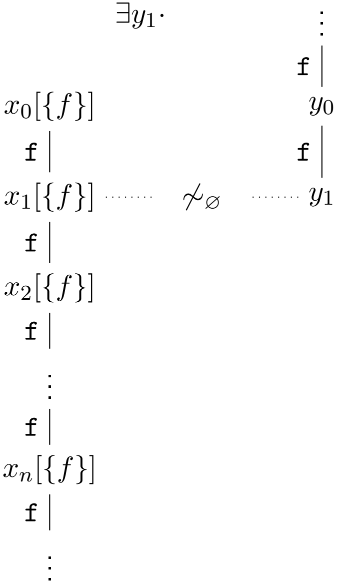
- R-NSIM-FENCE with x_0 and y_0;
- S-FEATS with x_1 and z_1

Does that even terminate?



R-NSIM-FENCE (for $F = \{f\}$ and $G = \varnothing$):

$$x[\{f\}] \wedge x \not\sim_\varnothing y \wedge c$$
$$\Rightarrow \quad \exists z, z' \cdot x[f]z \wedge y[f]z' \wedge z \not\sim_\varnothing z' \wedge x[\{f$$

- R-NSIM-FENCE with x_0 and y_0;
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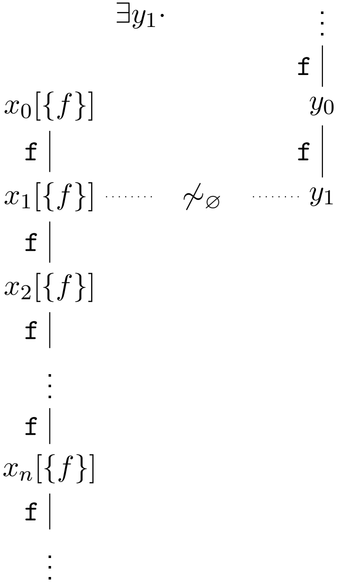
Does that even terminate?

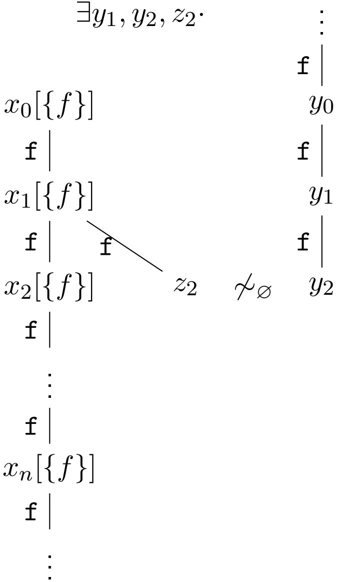


R-NSim-Fence (for $F = \{f\}$ and $G = \varnothing$):

$$x[\{f\}] \wedge x \not\sim_\varnothing y \wedge c$$
$$\Rightarrow \quad \exists z, z' \cdot x[f]z \wedge y[f]z' \wedge z \not\sim_\varnothing z' \wedge x[\{f$$

- R-NSim-Fence with x_0 and y_0;
- S-Feats with x_1 and z_1
- R-NSim-Fence with x_1 and y_1;

Does that even terminate?



R-NSIM-FENCE (for $F = \{f\}$ and $G = \varnothing$):

$$x[\{f\}] \wedge x \not\sim_\varnothing y \wedge c$$
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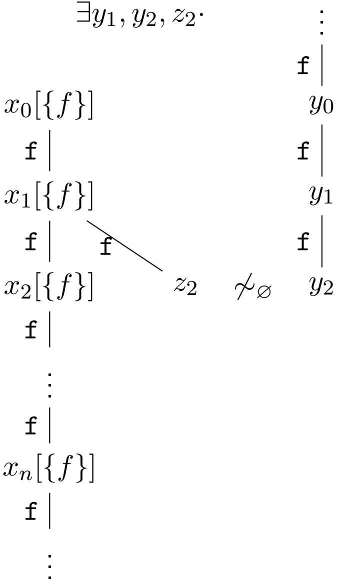
Does that even terminate?



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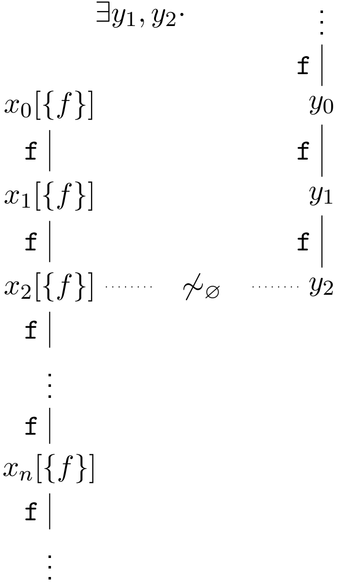

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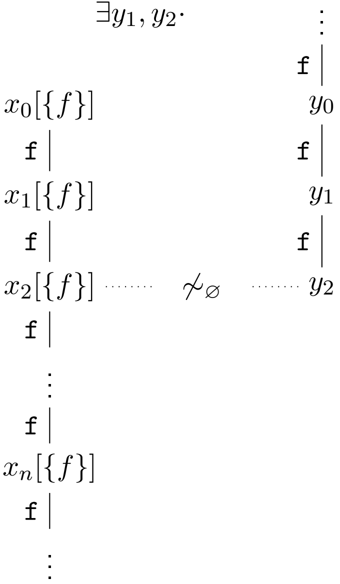

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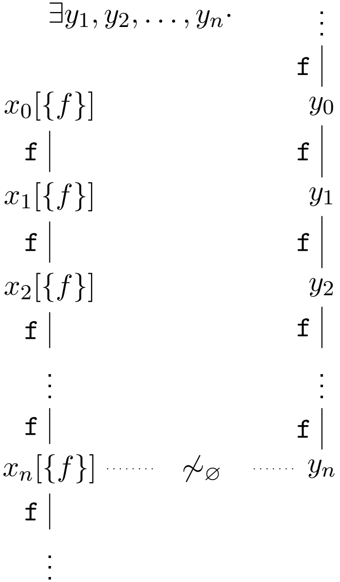

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Demo!